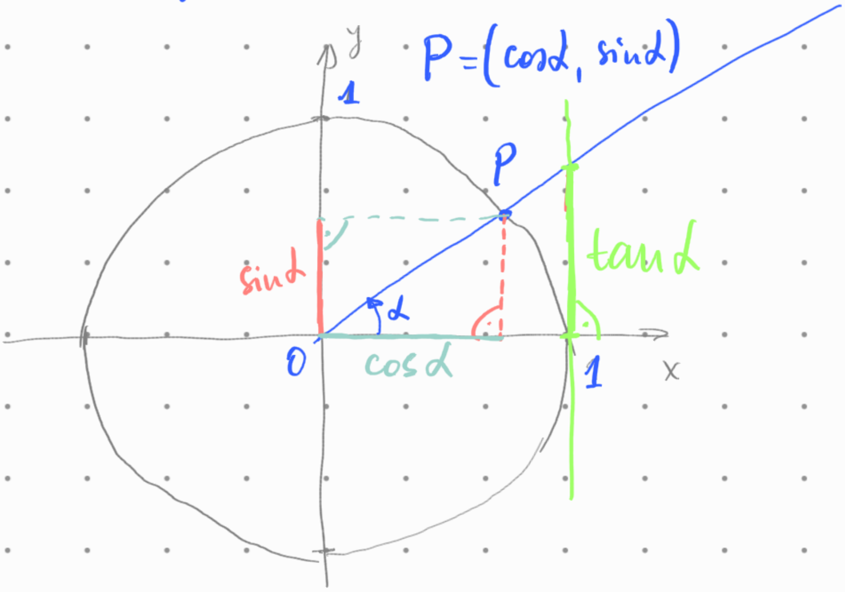
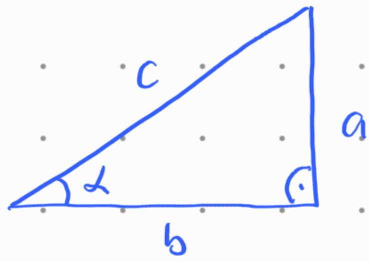
Trigonometrie





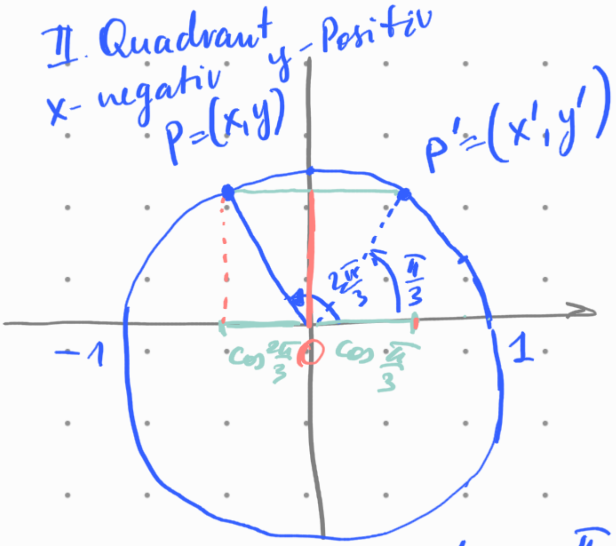
$$\sin\alpha = \frac{a}{c}\,;\ \cos\alpha = \frac{b}{c}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{a}{b}$$

Bsp $\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

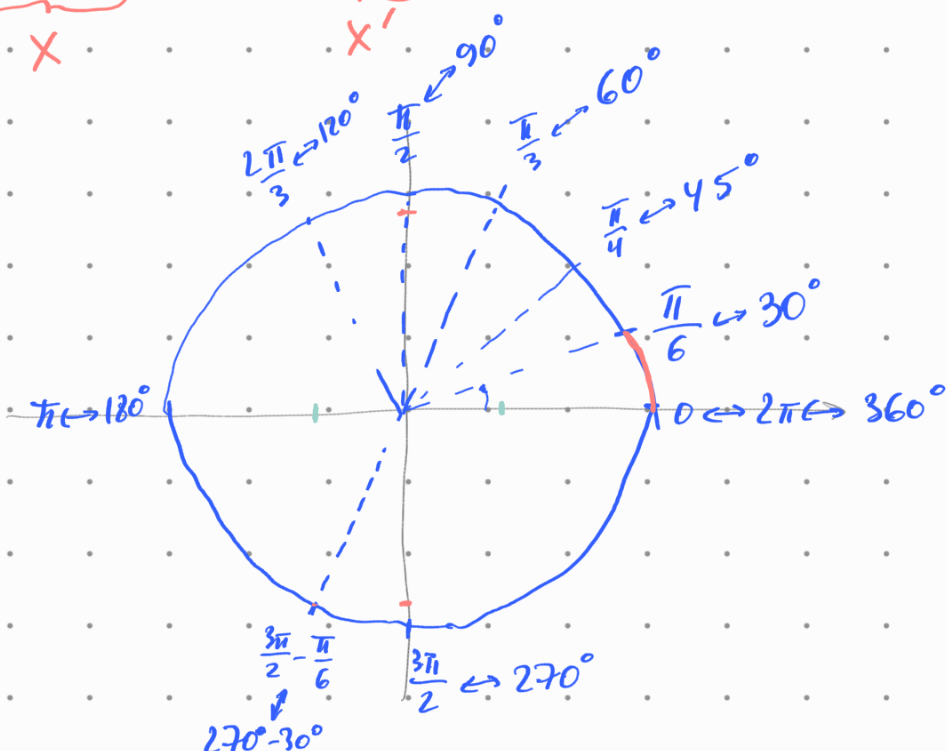
α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\alpha$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—

$$\underbrace{\cos\frac{2\pi}{3}}_{x} = -\underbrace{\cos\frac{\pi}{3}}_{x'} = -\frac{1}{2}$$



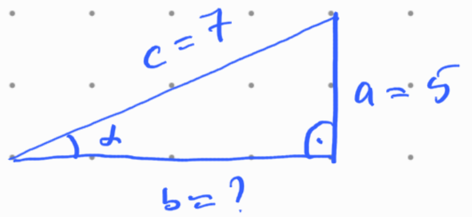
$x = \cos\frac{2\pi}{3}$ $\quad x' = \cos\frac{\pi}{3}$

$\boxed{|x| = |x'|}$, $x = -x'$



Bsp $\sin\alpha = \frac{5}{7}$ wobei $\left(0 \le \alpha \le \frac{\pi}{2}\right)$

Berechne $\cos\alpha$ und $\tan\alpha$.



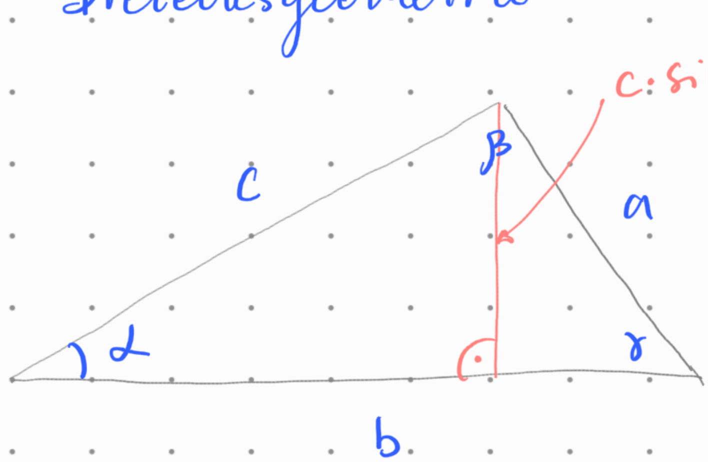
$$\sin\alpha = \frac{5}{7}$$

$$\cos\alpha = \frac{b}{c} = \frac{2\sqrt{6}}{7}$$

$$\tan\alpha = \frac{a}{b} = \frac{5\cdot\sqrt{6}}{2\sqrt{6}\cdot\sqrt{6}} = \frac{5\sqrt{6}}{12}$$

$$b = \sqrt{c^2 - a^2} = \sqrt{49-25}$$
$$= \sqrt{24} = \sqrt{4\cdot 6} = 2\sqrt{6}$$

➡ Dreiecksgeometrie



$$\boxed{\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}} = \frac{b}{\sin\beta}$$

Sinussatz

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Cosinussatz

Flächeninhalt $= \frac{1}{2}$ Grundseite · Höhe

(z.B.) $= \frac{1}{2} bc \sin\alpha$

Bsp $\alpha = \frac{\pi}{6}$, $a = 1$, $c = 2$. Finde alles andere.

1) Sinussatz: $\sin\gamma = \frac{c \cdot \sin\alpha}{a} = \frac{2\cdot\sin\frac{\pi}{6}}{1}$

$$= \frac{2\cdot\frac{1}{2}}{1} = 1$$

$\sin\gamma = 1 \Rightarrow \gamma = \frac{\pi}{2}$ $\quad\left(\frac{\pi}{2} \leftrightarrow 90°\right)$

2) Winkelsumme: $\beta = \pi - \alpha - \gamma = \pi - \frac{\pi}{6} - \frac{\pi}{2} = \frac{\pi}{3}$

3) Rechtwinklig! Pythagoras: $b^2 = c^2 - a^2$

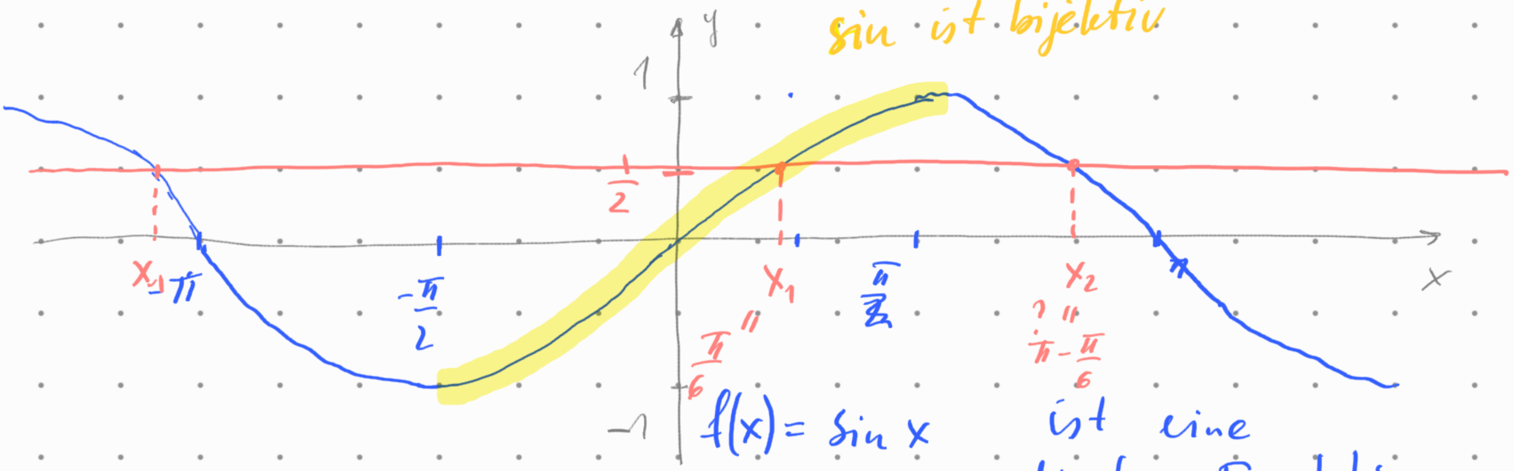
$= 4 - 1 = 3 \Rightarrow b = \sqrt{3}$

i.A. Cosinussatz $b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$

$b^2 = 1 + 4 - 2 \cdot 1 \cdot 2 \cdot \frac{1}{2} = 5 - 2 = 3$

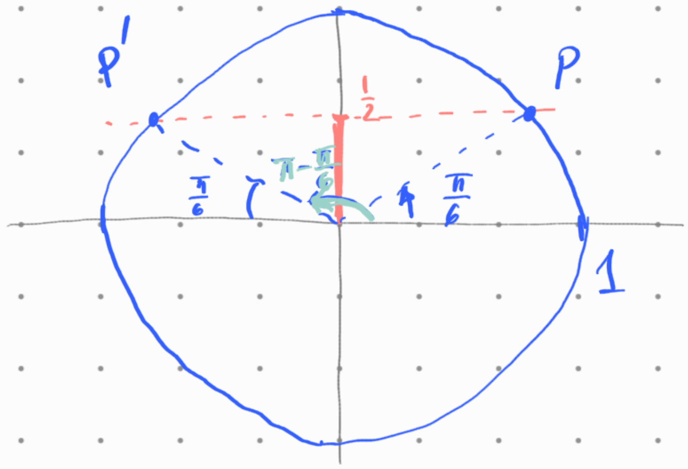
▶ Trigonometrische Gleichungen

Bsp 1) Bestimme alle Lösungen von $\sin x = \frac{1}{2}$



$f(x) = \sin x$ ist eine 2π-periodische Funktion

Finde die Lösungen $0 \leq x \leq 2\pi$.
Die anderen Lösungen wiederholen sich ...



1) also gilt $\sin x = \frac{1}{2}$

für $x_1 = \frac{\pi}{6}$

und $x_2 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

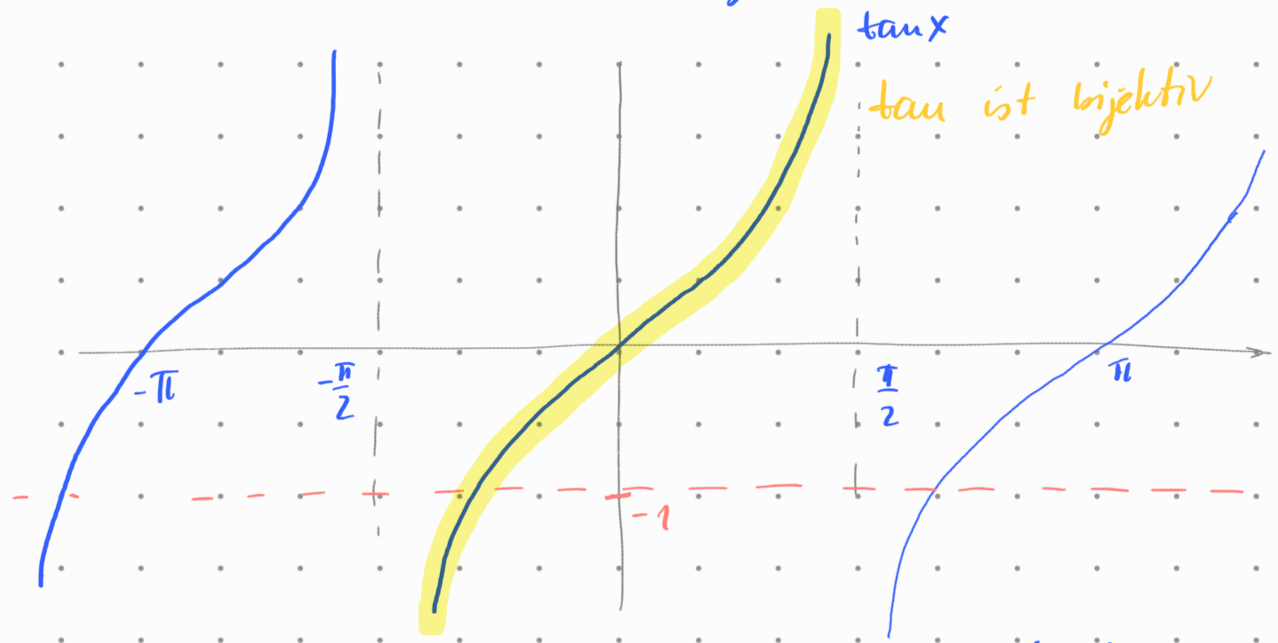
die beiden Lösungen zwischen 0 und 2π

2) Alle Lösungen:

$$\begin{cases} x = \frac{\pi}{6} + 2\pi k, & k \in \mathbb{Z} \\ x = \frac{5\pi}{6} + 2\pi k, & k \in \mathbb{Z} \end{cases}$$

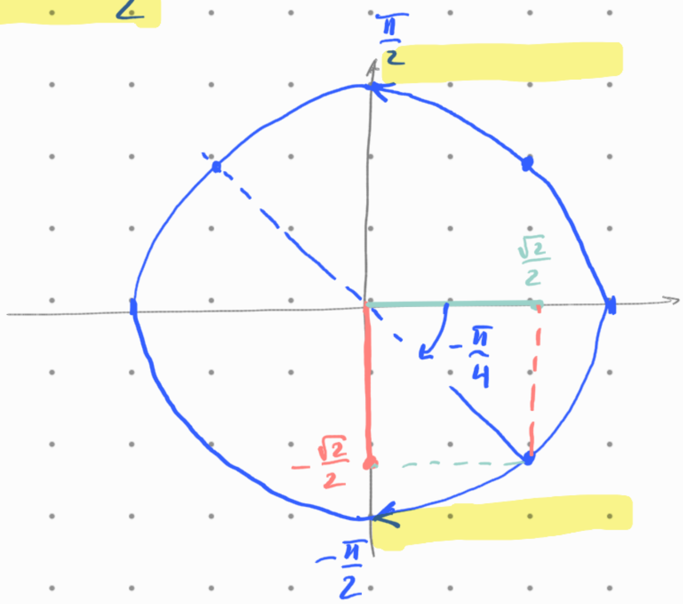
(beliebige ganze Zahl)

2) Finde alle Lösungen von $\tan x = -1$.



$f(x) = \tan x$ ist π-periodisch

1) Finde die Lösung $-\frac{\pi}{2} < x < \frac{\pi}{2}$
2) Sie wiederholt sich ...



1) $\tan x = -1 \iff \frac{\sin x}{\cos x} = -1$

$\sin x = -\cos x$

$\cos x > 0$
$\Rightarrow \sin x < 0$

$\Rightarrow x = -\frac{\pi}{4}$

2) alle Lösungen:

$x = -\frac{\pi}{4} + \pi k, \quad k \in \mathbb{Z}$

▶ Arcus-Funktionen

$\sin x = \frac{1}{2} \quad \left(-\frac{\pi}{2} \le x \le \frac{\pi}{2}\right) \rightsquigarrow x = \arcsin\left(\frac{1}{2}\right)$

Arcussinus, die zu sinus inverse Funktion

$\tan x = -1 \quad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right) \longleftrightarrow x = \arctan(-1)$

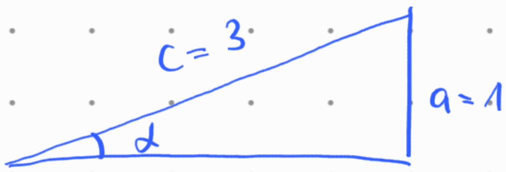
Bsp 1) $\arcsin(-1) = -\frac{\pi}{2}$,

denn $\sin\left(-\frac{\pi}{2}\right) = -\sin\frac{\pi}{2} = -1$

$\sin\alpha = \frac{1}{3}$

2) Berechne exakt $\cos(\underbrace{\arcsin \tfrac{1}{3}}_{\alpha})$

Bekannt: $\sin\alpha = \frac{1}{3}$, Finde $\cos\alpha$.



$b = \sqrt{c^2 - a^2} = \sqrt{9-1}$
$= \sqrt{8} = 2\sqrt{2}$

$$\Rightarrow \cos\alpha = \frac{b}{c} = \frac{2\sqrt{2}}{3}$$

$$\cos\alpha = \cos\left(\arcsin\tfrac{1}{3}\right) = \frac{2\sqrt{2}}{3}$$